

Activity P22: Rotational Inertia (Rotary Motion Sensor)

Concept	<i>DataStudio</i>	<i>ScienceWorkshop</i> (Mac)	<i>ScienceWorkshop</i> (Win)
Rotational motion	P22 Rotational Inertia.DS	(See end of activity)	(See end of activity)

Equipment Needed	Qty	Equipment Needed	Qty
Rotary Motion Sensor (CI-6538)	1	Mass and Hanger Set (ME-9348)	1
Balance (SE-8723)	1	Rotational Accessory (CI-6691)	1
Base and Support Rod (ME-9355)	1	Thread (inc. w/ CI-6691)	1 m
Calipers (SF-8711)	1		

What Do You Think?

What is an example of an object or device that depends on rotational inertia? How does the rotational inertia of a ring (or hoop) compare to the rotational inertia of a disk (or cylinder)?



Take time to answer the 'What Do You Think?' question(s) in the Lab Report section.

Background



A quarterback on an American football team throws the ball so it spirals in flight. A figure skater performs an elegant spin on the ice and increases her rotation rate by moving her outstretched arms closer to her body. Rotational inertia plays an important role in both of these phenomena.



The rotational inertia of an object depends on the mass and the distribution of mass. In general, the more compact an object, the less rotational inertia it has.

Theoretically, the rotational inertia, I , of a ring is given by

$$I = \frac{1}{2} M(R_1^2 + R_2^2) \quad \text{Equation 1}$$

where M is the mass of the ring, R_1 is the inner radius of the ring, and R_2 is the outer radius of the ring.

Theoretically, the rotational inertia, I , of a solid disk of uniform density is given by

$$I = \frac{1}{2} MR^2 \quad \text{Equation 2}$$

where M is the mass of the disk, R is the radius of the disk.

To find the rotational inertia of the ring and disk experimentally, apply a known torque to the ring and disk and measure the resulting angular acceleration.

Since $\tau = I\alpha$

$$I = \frac{\tau}{\alpha} \quad \text{Equation 3}$$

where α is the angular acceleration and τ is the torque.

Torque depends on the force applied and the distance from the pivot point of the rotating object to the point where the force is applied, or

$$\tau = \mathbf{r} \times \mathbf{F} \text{ Equation 4}$$

where \mathbf{r} is the distance from the center of the ring or disk to the point where a force is applied (the 'lever arm'), and \mathbf{F} is the applied force. The value of $\mathbf{r} \times \mathbf{F}$ is $r F \sin \phi$ where ϕ is the angle between \mathbf{r} and the direction of \mathbf{F} , the applied force. The torque is maximum when \mathbf{r} and \mathbf{F} are perpendicular.

In this case, the applied force is the tension (\mathbf{T}) in a thread that is attached to a part of a rotational apparatus. Gravity pulls a hanging mass \mathbf{m} that is attached to the thread. The value of \mathbf{r} is the radius of the step pulley on the apparatus. The radius is perpendicular to the applied force (Tension).

Therefore, the torque is:

$$\tau = rT \text{ Equation 5}$$

The following solution is derived from the convention that up is positive and down is negative, counter-clockwise is positive and clockwise is negative.

Applying Newton's Second Law for the hanging mass, \mathbf{m} , results in:

$$\sum \mathbf{F} = \mathbf{T} - \mathbf{mg} = \mathbf{m}(-\mathbf{a})$$

Solving for the tension in the string gives

$$\mathbf{T} = \mathbf{m}(\mathbf{g} - \mathbf{a})$$

The torque is:

$$\tau = rT = rm(\mathbf{g} - \mathbf{a}) \text{ Equation 6}$$

The linear acceleration \mathbf{a} of the hanging mass is the tangential acceleration, \mathbf{a}_T , of the rotating apparatus.

The angular acceleration is related to the tangential acceleration as follows:

$$\alpha = \frac{\mathbf{a}_T}{r} \text{ Equation 7}$$

Substituting Equation 6 and Equation 7 into Equation 3 gives:

$$\mathbf{I} = \frac{\tau}{\alpha} = rm(\mathbf{g} - \mathbf{a}) \div \frac{\mathbf{a}_T}{r} = rm(\mathbf{g} - \mathbf{a}) \frac{r}{\mathbf{a}_T} = \frac{mgr^2}{\mathbf{a}_T} - mr^2 = mr^2 \left(\frac{\mathbf{g}}{\mathbf{a}_T} - 1 \right)$$

The system's rotational inertia, \mathbf{I} , can be calculated from the tangential acceleration, \mathbf{a}_T .

For You To Do

Measure the mass and dimensions of a ring and a disk and calculate the theoretical values of rotational inertia. Use the Rotary Motion Sensor to measure the motion of a hanging mass that is connected by a thread to a step pulley on the rotational apparatus. Use *DataStudio* or *ScienceWorkshop* to record and display velocity versus time. The slope of velocity versus time is the tangential acceleration.

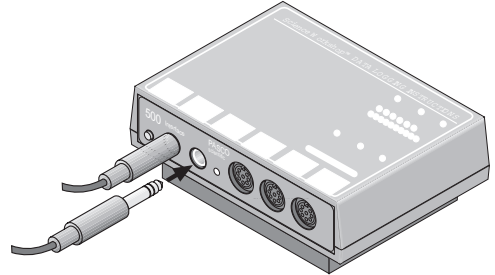
SAFETY REMINDERS

- Follow directions for using the equipment.

THINK SAFETY
ACT SAFELY
BE SAFE!

PART I: Computer Setup

1. Connect the *ScienceWorkshop* interface to the computer, turn on the interface, and turn on the computer.
2. Connect the Rotary Motion Sensor's stereo phone plugs into Digital Channels 1 and 2 on the interface. Remove the 'O' ring from the large groove on the step pulley on the sensor.
3. Open the file titled as shown:



<i>DataStudio</i>	<i>ScienceWorkshop</i> (Mac)	<i>ScienceWorkshop</i> (Win)
P22 Rotational Inertia.DS	(See end of activity)	(See end of activity)

- The *DataStudio* file has a Workbook display and a Graph display. Read the instructions in the workbook.
- Data recording is set at 20 Hz. The Rotary Motion Sensor is set for 360 divisions per rotation. The 'Linear Calibration' is set for 'Large Groove (Pulley)'.
- See the pages at the end of this activity for information about modifying a *ScienceWorkshop* file.

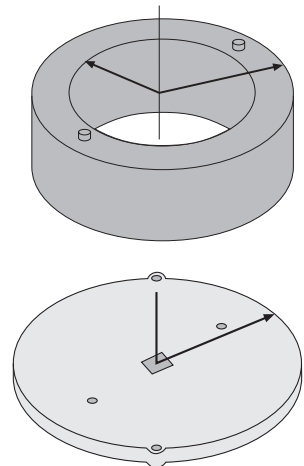
PART II: Sensor Calibration and Equipment Setup

- You do not need to calibrate the sensor.

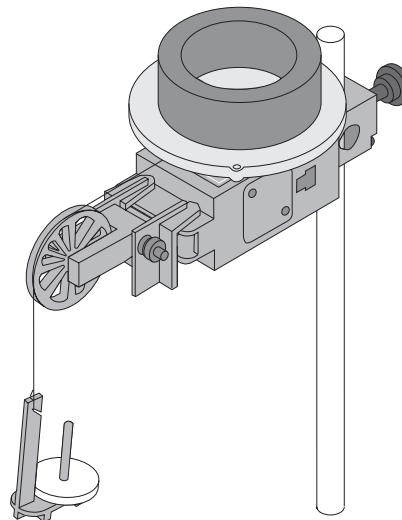
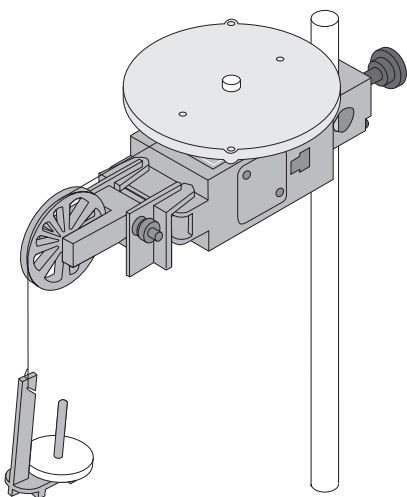
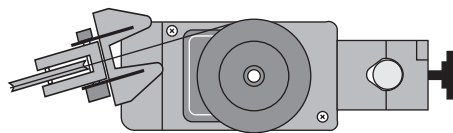
Equipment Setup

For this part you will need the following: ring, disk, clamp-on Super Pulley, thread (all part of the Rotational Accessory), calipers, balance, base and support rod, Rotary Motion Sensor, mass hanger.

1. Complete **DataTable #1** in the Lab Report section.
 - Measure the diameter of the large groove on the step pulley on Rotary Motion Sensor. Calculate and record the radius of the step pulley.
 - Measure the inside and outside diameters of the ring. Calculate and record the inside radius and the outside radius of the ring.
 - Measure and record the radius of the disk.
 - Measure and record the mass of the ring and the mass of the disk.
2. Mount the Rotary Motion Sensor on a support rod so the step pulley is on top.
3. Mount the clamp-on Super Pulley on the end of the Rotary Motion Sensor.



- Use a piece of thread about 10 cm longer than the distance from the Super Pulley to the ground. Tie one end of the thread to the edge of the large groove on the step pulley on the Rotary Motion Sensor. Drape the thread over the Super Pulley. Tie the other end of the thread to a mass hanger. Adjust the angle of the Super Pulley so the thread is tangent to the step pulley and in the middle of the groove on the Super Pulley.
- Remove the thumbscrew from the step pulley on top of the Rotary Motion Sensor. Place the disk on the pulley and attach the disk with the thumbscrew.
- Place the ring on the disk inserting the ring pins into the holes on the disk.



PART IIIA: Data Recording – Acceleration of the Ring and Disk

Measure the acceleration of the ring and disk together.


- Add about 20 g to the mass hanger attached to the thread. Wind the thread around the large groove on the step pulley of the Rotary Motion Sensor by rotating the disk until the mass hanger is raised almost to the Super Pulley. Hold the disk in place.
- Start recording data and release the disk.
- Stop recording data just before the mass hanger reaches the floor.
 - “Run #1” will appear in the Data list in the Experiment Setup window.
- Remove the mass hanger from the thread. Measure the total mass of the hanger and record the value in **Data Table 2**.

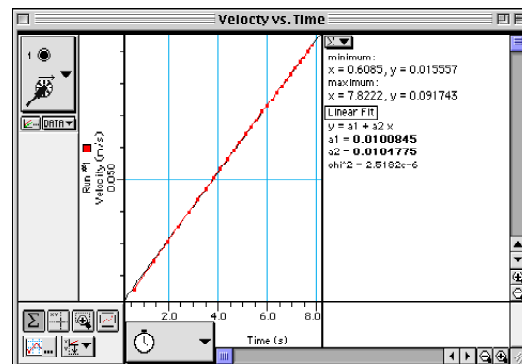
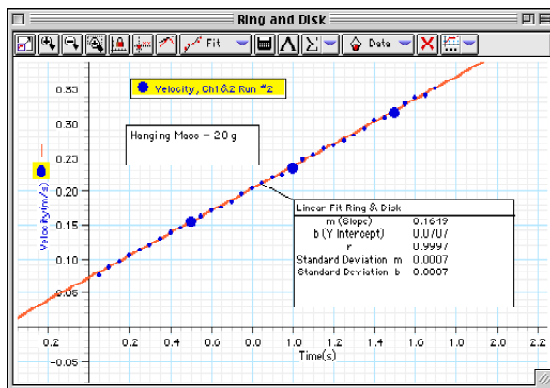
PART IIIB: Data Recording – Acceleration of the Disk Alone

Measure the acceleration of the disk alone.



- Remove the ring from the disk. Add about 20 g to the mass hanger attached to the thread. Wind the thread around the large groove on the step pulley of the Rotary Motion Sensor by rotating the disk until the mass hanger is raised almost to the Super Pulley. Hold the disk in place.
- When you are ready, start recording data and then release the disk.
- Stop recording data just before the mass and hanger reaches the floor.
 - “Run #2” will appear in the Data list in the Experiment Setup window.
- Remove the mass hanger from the thread. Measure the total mass of the hanger and record the value in the **Data Table 2**.

Analyzing the Data

- Use the Graph display’s built-in analysis tools to find the slope of the velocity versus time.
 - In *DataStudio*, use the cursor to draw a rectangle around the region of smoothest data in the Graph. Click the ‘Fit’ menu in the Graph toolbar and select ‘Linear’.
 - In *ScienceWorkshop*, click the ‘Statistics’ button () to open the Statistics area on the right side of the Graph. Rescale the display. Use the cursor to draw a rectangle around the region of smoothest data. Select ‘Curve Fit, Linear Fit’ from the Statistics menu.



- Record the acceleration for both Run #1 (Ring and Disk) and Run #2 (Disk alone).

- Hint: Use the ‘Data’ menu ( or ) in the Graph display to select Run #2.
- Calculate and record the experimental rotational inertia, I , of the Ring and Disk using the measured acceleration “ a_T ”, the Step Pulley radius “ r ”, and the mass “ m ” that caused the apparatus to rotate.

$$I = \frac{\tau}{\alpha} = mr^2 \left(\frac{g}{a_T} - 1 \right)$$

- Calculate and record the experimental rotational inertia of the Disk using the measured acceleration “ a_T ”, the Step Pulley radius “ r ”, and the mass “ m ”.

5. Subtract the rotational inertia of the Disk from the value of the rotational inertia for the Ring and Disk to find the rotational inertia of the Ring alone. Record the experimental value for the Ring.
6. Calculate the theoretical value of the rotational inertia of the Ring based on its dimensions (R_1 and R_2) and mass (M). Record the value.

$$I = \frac{1}{2}M(R_1^2 + R_2^2)$$

7. Calculate the theoretical value of the rotational inertia of the Disk based on its dimension (R) and mass (M). Record the value.

$$I = \frac{1}{2}MR^2$$

8. Calculate the percent difference between the experimental values and the theoretical values for the rotational inertia of the Ring and the rotational inertia of the disk.

Record your results in the Lab Report section.

Lab Report – Activity P22: Rotational Inertia**What Do You Think?**

What is an example of an object or device that depends on rotational inertia? How does the rotational inertia of a ring (or hoop) compare to the rotational inertia of a disk (or cylinder)?

Data Table #1: Dimensions

Item	Value
Step Pulley radius (r)	
Ring, inside radius (R_1)	
Ring, outside radius (R_2)	
Disk, radius (R)	
Mass of Ring (M)	
Mass of Disk (M)	

Data Table #2: Mass

Run	Description	Hanging Mass
#1	Ring and Disk	
#2	Disk Alone	

Data Table #3: Measured Accelerations (a_T)

Run	Description	Acceleration
#1	Ring and Disk	
#2	Disk Alone	

Data Table #4: Rotational Inertia (I)

Run	Description	Experimental	Theoretical	% Difference
#1	Ring and Disk			
#2	Disk Alone			
	Ring Alone			

Questions

1. How do the experimental values compare to the theoretical values for rotational inertia?
2. What are some reasons that could account for any differences?

Appendix: Modify a *ScienceWorkshop* File

Modify an existing *ScienceWorkshop* file to add the Rotary Motion Sensor.

Open the *ScienceWorkshop* File

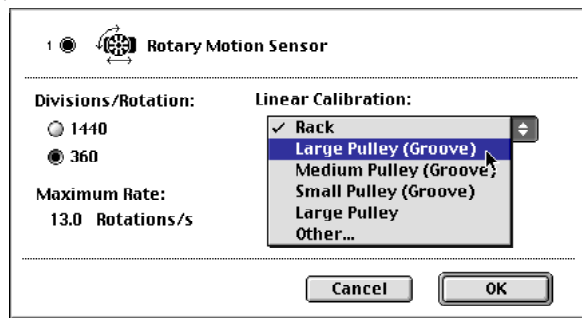
Open the file titled as shown:

<i>ScienceWorkshop</i> (Mac)	<i>ScienceWorkshop</i> (Win)
P26 Rotational Inertia	P26_ROT.A.SWS

- The *ScienceWorkshop* file has a Graph display of Velocity versus Time (using data from a Smart Pulley).
- You need to replace the Smart Pulley with the Rotary Motion Sensor in the Experiment Setup window and set up the Rotary Motion Sensor parameters. Then you need to change the Graph display.

Set Up the Rotary Motion Sensor

In the Experiment Setup window, click and drag the digital sensor plug to Channel 1. Select 'Rotary Motion Sensor' from the list of sensors. Click 'OK'. **Result:** The Rotary Motion Sensor setup window opens.



Under Linear Calibration, select 'Large Pulley (Groove)'. Click 'OK' to return to the Experiment Setup window.

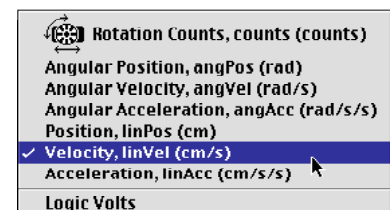
Set the Sampling Options

Click the 'Sampling Options' button in the Experiment Setup window or select 'Sampling Options' from the Experiment menu to open the Sampling Options window. **Result:** The Sampling Options window opens.

Under 'Periodic Samples' click the right arrow to set the sample rate at '20 Hz' (20 measurements per second). Click 'OK' to return to the Experiment Setup window.

Change the Graph Display

Click the 'Y-Axis Input' menu and select 'Digital 1, Velocity, linVel (cm/s)'.



Activity P23: Conservation of Angular Momentum (Rotary Motion Sensor)

Concept	<i>DataStudio</i>	<i>ScienceWorkshop</i> (Mac)	<i>ScienceWorkshop</i> (Win)
Rotational motion	P23 Angular Momentum.DS	(See end of activity)	(See end of activity)

Equipment Needed	Qty	Equipment Needed	Qty
Rotary Motion Sensor (CI-6538)	1	Calipers (SF-8711)	1
Balance (SE-8723)	1	Rotational Accessory (CI-6691)	2
Base and Support Rod (ME-9355)	1		

What Do You Think?

What interesting sporting events can be understood using the concept of conservation of angular momentum?



Take time to answer the 'What Do You Think?' question(s) in the Lab Report section.

Background

When a net torque τ is applied to an object that is free to rotate, there is a change in the angular momentum (ΔL) of the object.

$$\tau = \frac{\Delta L}{\Delta T}$$

When a non-rotating disk is dropped on a rotating disk, there is no net torque on the system since the torque on the non-rotating is equal and opposite to the torque on the rotating disk. If there is no change in angular momentum the angular momentum is conserved:

$$L = I_i \omega_i = I_f \omega_f$$

where I_i is the initial rotational inertia and ω_i is the initial rotational speed. The initial rotational inertia is that of a disk.

$$\frac{1}{2} MR^2$$

If the second disk has the same rotational inertia as the first disk, the final rotational inertia is twice the initial rotational inertia of the first disk.

If angular momentum is conserved, the final rotational speed is half of the initial rotational speed:

$$\omega_f = \frac{I_i}{I_f} \omega_i = \frac{1}{2} \omega_i$$

SAFETY REMINDERS

- Follow directions for using the equipment.

THINK SAFETY
ACT SAFELY
BE SAFE!

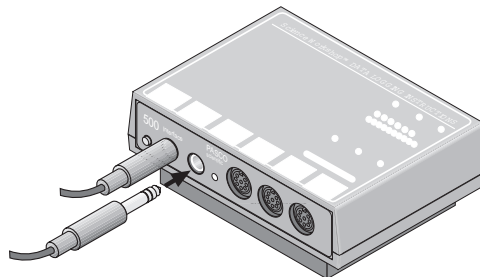
For You To Do

The goal in this activity is to measure the final angular speed of a system consisting of a non-rotating ring that is dropped onto a rotating disk and to compare the measured angular speed to the value predicted using conservation of angular momentum.

Use the Rotary Motion Sensor to measure the angular speed of a rotating disk before and after a second disk that is not rotating is dropped onto the rotating disk. Use *DataStudio* or *ScienceWorkshop* to record and display the angular speed before and after the torque-free collision.

PART I: Computer Setup

1. Connect the *ScienceWorkshop* interface to the computer, turn on the interface, and turn on the computer.
2. Connect the Rotary Motion Sensor's stereo phone plugs into Digital Channels 1 and 2 on the interface. Remove the 'O' ring from the large groove on the step pulley on the sensor.
3. Open the file titled as shown:

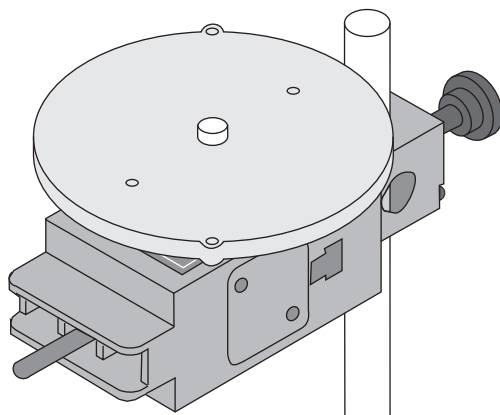


<i>DataStudio</i>	<i>ScienceWorkshop</i> (Mac)	<i>ScienceWorkshop</i> (Win)
P23 Angular Momentum.DS	(See end of activity)	(See end of activity)

- The *DataStudio* file has a Workbook display and a Graph display. Read the instructions in the workbook.
- Data recording is set at 20 Hz. The Rotary Motion Sensor is set for 360 divisions per rotation.
- See the pages at the end of this activity for information about modifying a *ScienceWorkshop* file.

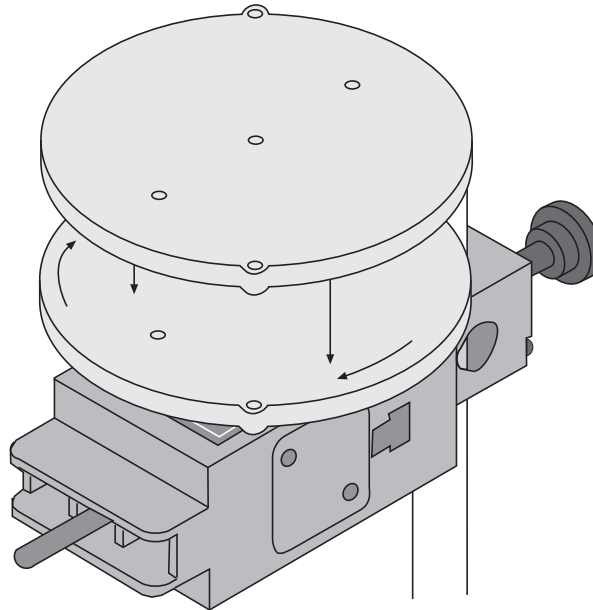
PART II: Sensor Calibration and Equipment Setup

- You do not need to calibrate the sensor.
1. Mount the Rotary Motion Sensor on a support rod so the step pulley is on top.
 2. Remove the thumbscrew from the step pulley on top of the Rotary Motion Sensor. Place the disk on the pulley and attach the disk with the thumbscrew.



PART III: Data Recording

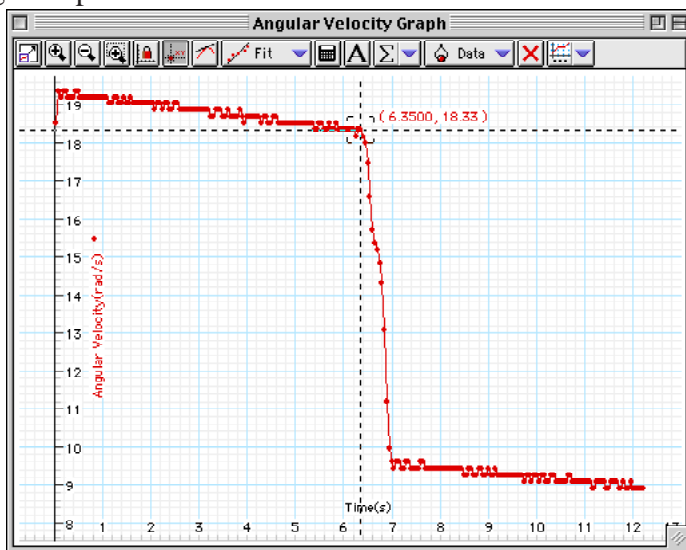
- In this part of the activity you will drop the second disk onto the disk that is attached to the Rotary Motion Sensor. Hold the second disk so the square indent on one side of the disk is above the thumbscrew on the first disk.
1. Give the first disk a spin using your hand.
 2. Start recording data. (Hint: Click 'Start' in *DataStudio* or 'REC' in *ScienceWorkshop*.)
 3. After about 25 data points have been recorded, drop the second disk onto the spinning one.



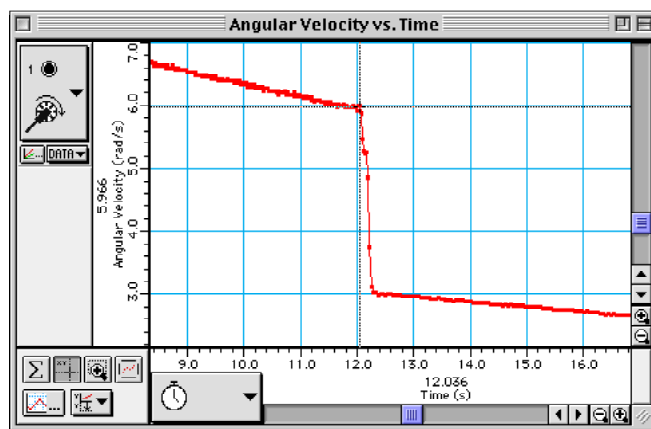
4. After another 25 or so data points have been recorded, stop data recording.
- Run #1 will appear in the Data list.
5. Repeat the data recording procedure a total of three times.

Analyzing the Data

- Use the Graph display's built-in analysis tools to determine the angular speed just before the disk was dropped and the angular speed just after the disk was dropped. Record the initial angular speed and the final angular speed in the Lab Report.
- In *DataStudio*, use the 'Smart Tool' to find the coordinates of the point on the plot just before the second disk was dropped on the first disk. Then find the coordinates of the point just after the second disk was dropped and the two disks are rotating together. The 'Y-coordinate' is the angular speed.



- In *ScienceWorkshop*, use the 'Smart Cursor' to find the coordinates of the point on the plot just before the second disk was dropped on the first disk. Then find the coordinates of the point just after the second disk was dropped and the two disks are rotating together. The 'Y-coordinate' is the angular speed.



- Calculate the expected (theoretical) values for the final angular speed ($\omega_f = 0.5 \omega_i$) and record it in the Lab Report.
- Repeat the data analysis process for each run, selecting the next run from the data list. Complete the data table by calculating the percent difference between the actual and the theoretical values of the final angular speed.

Record your results in the Lab Report section.

Lab Report – Activity P23: Conservation of Angular Momentum**What Do You Think?**

What interesting sporting events can be understood using the concept of conservation of angular momentum?

Data Table

Trial	ω_i (rad/s)	ω_f (actual) (rad/s)	ω_f (theory) (rad/s)	% Difference
Run #1				
Run #2				
Run #3				

Questions

1. How does the experimental value for the final angular velocity agree with the theoretical value for the final angular velocity?

2. What are possible reasons for the difference between the experimental value and the theoretical value, if any?

Appendix: Modify a *ScienceWorkshop* File

Modify an existing *ScienceWorkshop* file to add the Rotary Motion Sensor.

Open the *ScienceWorkshop* File

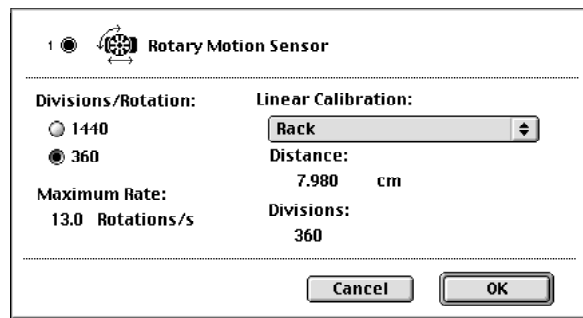
Open the file titled as shown:

<i>ScienceWorkshop</i> (Mac)	<i>ScienceWorkshop</i> (Win)
P27 Cons. Angular Momentum	P27_ANGU.SWS

- The *ScienceWorkshop* file has a Graph display of Angular Velocity versus Time (using data from a Smart Pulley).
- You need to replace the Smart Pulley with the Rotary Motion Sensor in the Experiment Setup window and set up the Rotary Motion Sensor parameters. Then you need to change the Graph display.

Set Up the Rotary Motion Sensor

In the Experiment Setup window, click and drag the digital sensor plug to Channel 1. Select 'Rotary Motion Sensor' from the list of sensors. Click 'OK'. **Result:** The Rotary Motion Sensor setup window opens. Click 'OK' to return to the Experiment Setup window.



Set the Sampling Options

Click the 'Sampling Options' button in the Experiment Setup window or select 'Sampling Options' from the Experiment menu to open the Sampling Options window. **Result:** The Sampling Options window opens.

Under 'Periodic Samples' click the right arrow to set the sample rate at '20 Hz' (20 measurements per second). Click 'OK' to return to the Experiment Setup window.

Change the Graph Display

Click the 'Y-Axis Input' menu and select 'Digital 1, Angular Velocity, angVel (rad/s)'.

