The Work-Energy Theorem and The Law of Conservation of Mechanical Energy

Mechanics Applications:
Work, kinetic energy, potential energy, the Work-Energy Theorem and the law of conservation of mechanical energy. Energy dissipation.

DataStudio files: **WorkEnergyTheorem.ds**

Equipment List

INCLUDED:

1	Economy Force Sensor	CI-6746
1	PAStrack Basic System	ME-6962
1	Compact Cart Mass	ME-6755
1	Photogate Head	ME-9498A
1	Photogate Bracket	ME-9806
1	Super Pulley with Clamp	ME-9448B
1	Mass and Hanger Set	ME-8979
1	Braided Physics String	SE-8050

NOT INCLUDED, BUT NEEDED:

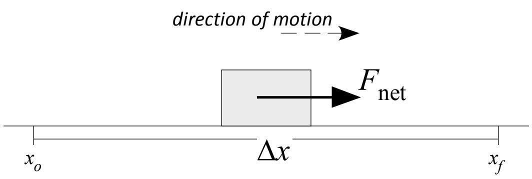
1	*ScienceWorkshop* 750 Interface	CI-7650
	DataStudio Software	
1	Balance	SE-8707

Introduction

The purpose of this activity is to compare the total work done on an object to the change in kinetic energy of the object. The force sensor is used to measure the force applied to a cart. The Photogate/Pulley System is used to measure the velocity of the cart as it is pulled by a hanging mass. *DataStudio* records and displays the force as a function of position and uses the velocity measurement to calculate the kinetic energy of the cart, of the hanging mass and of the system. The total work done, determined as the area under the Force vs. Position plot, is then compared to the change in kinetic energy. The analysis explores the effects of dissipative forces.

Theory

For an object with mass m that experiences a constant net force F_{net} over a displacement $\Delta x = x_f - x_o$ parallel to the net force, the total work done is:

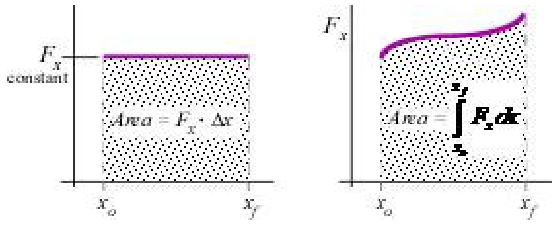


$$W_{\text{TOTAL}} = F_{\text{net}} \cdot \Delta x$$

The net force is the vector addition of all forces acting on the object during the displacement. If the net force varies during the displacement, then the total work done is calculated as an integral:

$$W_{\text{TOTAL}} = \int_{x_o}^{x_f} F_{\text{net}}\, dx\,.$$

Either way, the total work can also be determined as the area under the curve of a plot of net force vs. position, as illustrated.

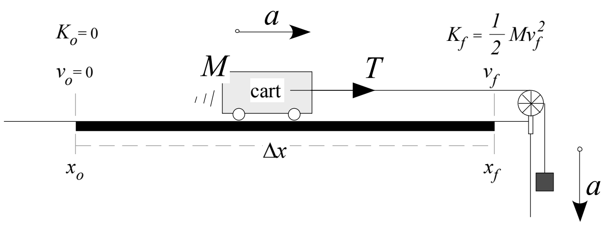


According to the Work-Energy Theorem, a change in kinetic energy can only be produced if work is done. The work done must be the combined effort of all forces involved (the net force), that is, the change in kinetic energy is given by the *total* amount of work done:

$$\Delta K = K_f - K_o = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2 = W_{\text{TOTAL}} = F_{\text{net(x)}} \cdot \Delta x$$ [Eq. 1 The Work-Energy Theorem]

The Cart of Mass M Pulled by the Hanging Mass m

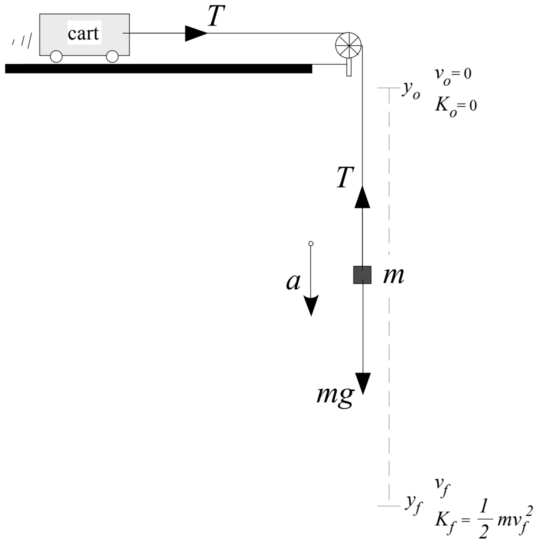
In this experiment, a cart will be pulled along a level track by a mass hanging from a string that passes over a pulley, as illustrated. The force sensor, mounted on the cart, will measure the pull from the string (the tensional force of the string, T in the diagram).



The cart will accelerate, starting from rest, up to a final speed v_f after undergoing a displacement Δx. Assuming there is no friction, then the tension on the string is also the net force that accelerates the cart, and the Work-Energy Theorem requires that

$$W_{\substack{\text{TOTAL}\\ \text{on cart}}} = T \cdot \Delta x = \Delta K_{\text{cart}}$$ [Eq. 2 The Total Work on the Cart.]

The Hanging Mass m that Pulls the Cart M



While the cart is moving horizontally on the track, the hanging mass is moving vertically. Because the cart and the hanging mass are coupled by the string, at any moment they have the same speed and the same acceleration. They also undergo the same total displacement, but for the hanging mass it is a vertical displacement, which means both its gravitational potential and its kinetic energy are changing as it moves. Notice that for the hanging mass, tension and gravity are both doing work as the mass moves.

The work done by gravity on the vertical motion of the hanging mass is:

$$W_{\text{by gravity}} = mg \cdot \Delta y$$

[Eq. 3 The Work done by Gravity.]

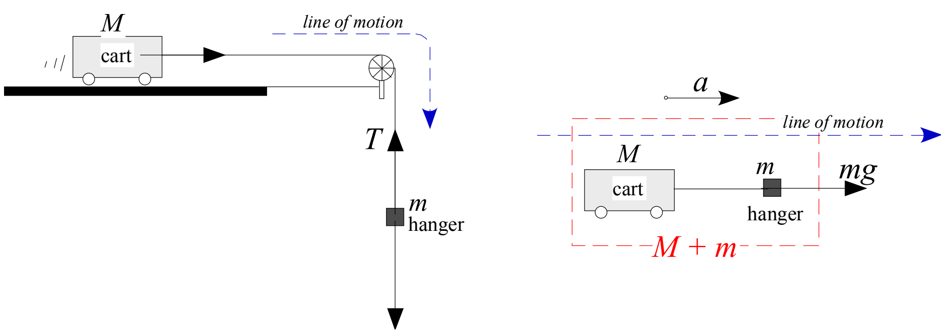
This is the negative of the change in potential energy of the hanging mass, that is, $\Delta U_{\text{hanger}} = -mg \cdot \Delta y$. (The negative arises because for an object moving vertically downward, potential energy decreases and $\Delta U < 0$ while gravity is doing positive work.) We can then determine the change in potential energy of the hanging mass as:

$$\Delta U_{\text{hanger}} = -W_{\text{by gravity}}$$

[Eq. 4 The Change in Potential Energy of the Hanging Mass.]

The Coupled System of Total Mass $M + m$

The cart and hanger system can be considered as if it were a single object of mass $M + m$ that moves with acceleration a along a line-of-motion that follows the string. The tensional force, being internal in this system, cannot provide the acceleration, and the net force on the combined system is just the weight of the hanging mass:



$$F_{\substack{\text{net}\\ \text{on system}}} = mg$$

For a displacement of magnitude $|\Delta x|$, the total work done on this system must be

$$W_{\substack{\text{TOTAL}\\ \text{on system}}} = F_{\substack{\text{net}\\ \text{on system}}} \cdot \Delta x = mg \cdot \Delta x\,.$$

Notice that this work has the same magnitude as the change in potential energy of the hanging mass, determined before. That is, $W_{\substack{\text{TOTAL}\\ \text{on system}}} = -\Delta U_{\text{hanger}}$.

The Work-Energy Theorem then requires that the change in kinetic energy of the coupled system be:

$$W_{\substack{\text{TOTAL}\\ \text{on system}}} = -\Delta U_{\text{hanger}} = \Delta K_{\text{system}}$$

This is a statement of the ***Law of Conservation of Mechanical Energy***: in the absence of dissipative forces, when a system undergoes a change in potential energy, there is a corresponding opposite change in the kinetic energy of the system. In this case, only the part of the system that moves vertically (the hanging mass) changes gravitational potential energy, but every part of the system changes kinetic energy.

$$-\Delta U_{\text{hanger}} = \Delta K_{\substack{\text{system of}\\ \text{hanger and cart}}}$$ [Eq. 5 The Conservation of Mechanical Energy in the System.]

SAFETY REMINDER	THINK SAFETY ACT SAFELY BE SAFE!
• Follow directions for using the equipment	

DATA STUDIO SET UP

1. Connect the Force Sensor and Photogate to the interface.

2. Open the *DataStudio* file: **WorkEnergyTheorem.ds.**

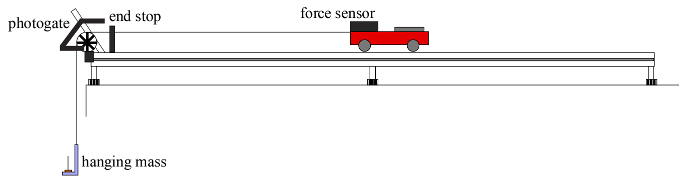
- The *DataStudio* file opens with graph displays of Position and vs. Time, Kinetic Energy vs. Time, and Tension vs. Position. The file performs calculations of kinetic energy based on the masses involved in the system and the measured velocities.
- The user must enter the masses of the cart and the hanging mass in order for DataStudio to perform the calculations. Instructions to do this are in the process.

EQUIPMENT SET UP

1. Use the thumbscrew that comes with the Economy Force Sensor to mount the sensor onto the accessory tray of a PAScar. Add the Compact Cart Mass to the tray of the PAScar.

2. Measure the mass of the cart with the force sensor and the extra mass. Record the value in kilograms in the Data Table as the 'Mass of the Cart,' (M_{cart}).

3. Place the track on a horizontal surface. Level the track, if needed: place the cart on the track and if the cart rolls one way or the other, use the leveling feet of the track to raise or lower that end until the track is level.

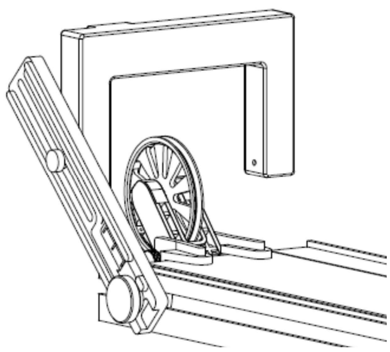


Note: It is very important that the track is level to get the best results.

4. Attach the Super Pulley to the end of the track.

5. Use the Photogate Bracket to attach the photogate head to the end of the track.

- Mount the bracket on the track so that the smooth side of the bracket is against the edge of the track.
- Slide the square nut into the T-slot of the track.
- Adjust the angle of the bracket so that the photogate head is looking at the spokes of the Super Pulley.



6. Place the end stop a few centimeters in front of the photogate and pulley.

7. Use a piece of string about 10 centimeters longer than the distance from the top of the pulley to the floor. Connect one end of a string to the force sensor's hook.

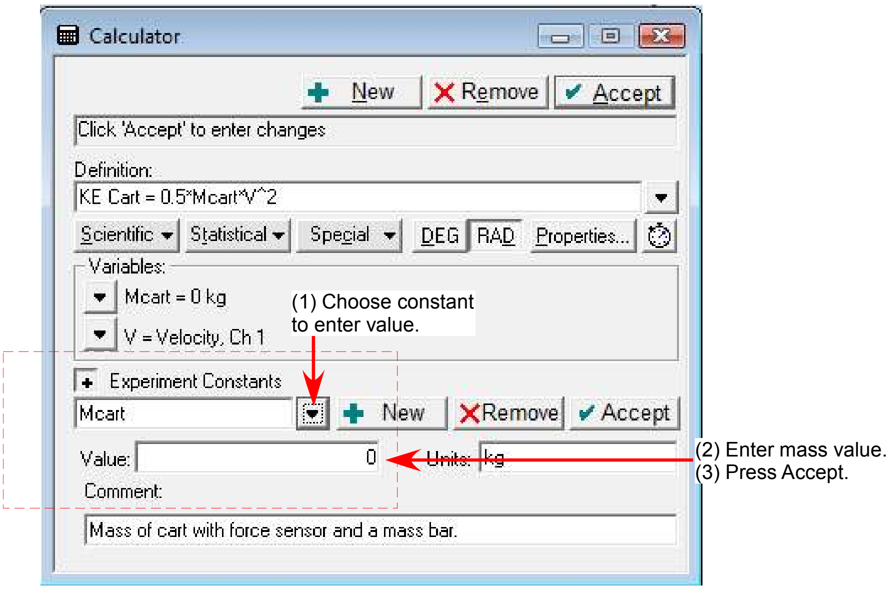
8. Press the 'Tare' button on the Force Sensor to zero the sensor. (The sensor needs to be zeroed when there is nothing pulling on it, so do it before installing the hanger over the pulley.)

9. Put a 20-g mass piece on a mass hanger. Weigh the hanger with the mass and record the value in kilograms in the Data Table as the 'Hanging Mass,' (m_{hanger}).

10. Attach the mass hanger to the end of the string and adjust the length of the string in such a way that the bottom of the hanger is just above the floor when the cart is against the end-stop.

11. Click the 'Calculate' button to open the Calculator window. Under the 'Experiment Constants' section of the Calculator window, enter the measured mass values for M_{cart} and m_{hanger} :

- Choose 'Mcart' from the drop menu.
- Enter the value in kilograms.
- Press 'Accept.'
- Repeat with 'mhanger.'



EXPERIMENTAL PROCEDURE:

- *Note: The procedure is easier to perform if one person handles the cart while another person handles the computer. Stop recording data before the cart hits the end stop.*

1. Pull the cart away from the Photogate so the hanging mass is just below the pulley.

2. Move the force sensor cable out of the way, if needed, so that it does not drag or interfere with the motion of the cart along the track.

3. Click ‘Start’. Release the cart.

4. Click ‘Stop’ just before the cart reaches the end stop.

5. Complete the Data Collection and the Analysis processes (next page) for this run before doing another one.

6. After completing the data collection and analysis for the first run, repeat the experiment two more times, adding some mass to the hanger each time.

- Record the new hanging mass in the Data Table.
- Access the Calculator and re-enter the experimental value of ‘m_{hanger}’ before collecting new data. (See Step 11 of the Setup.)
- Complete the data collection process for this run before changing the value of ‘m_{hanger}’ again.

DATA COLLECTION:

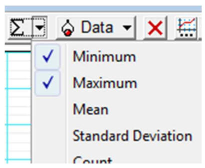
1. Examine the Position vs. Time plot:

- Record the initial (x_o) and final (x_f) positions of the cart by recording the 'Min' and 'Max' values given in the legend.
- Calculate the displacement of the car, $\Delta x = x_f - x_o$.

2. One at a time, examine the kinetic energy plots:

- Record the initial and final kinetic energies of the cart, the hanging mass and the system, by recording the the 'Min' and 'Max' values given in the legends.
- Calculate the change in kinetic energy for each case, $\Delta K = K_f - K_o$.
- Note: If the legend does not show the minimum and maximum values, click the Statistics Menu button to select them.

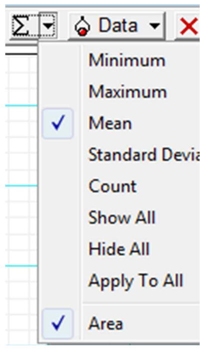
Statistics Menu



Choose to display minimum and maximum Kinetic Energy values.

3. Examine the Tension versus Position plot:

- With a click-and-drag motion of the mouse, select the region of the plot that corresponds to when the cart was moving. (From x_o to x_f, as recorded before.)
- After selecting the area of interest, record the 'Area' in the Analysis Table. This is the area under the curve.
- Record also the 'Mean' value of the tension in the Data Table. This is the average force that pulled on the cart during the motion.
- Note: If the legend does not show the area or the mean, click the Statistics Menu button to select them.



Display the area under the F vs. x plot and the average force.

ANALYSIS:

1. The Work-Energy Theorem:

- According to Eq. 2, the Work-Energy Theorem predicts that the total work done on the cart must be equal to the change in kinetic energy of the cart. Compare the total work done to the change in kinetic energy by taking a percent difference.

2. The Law of Conservation of Energy:

- Calculate the work done by gravity on the vertical motion of the hanger, using Eq. 3. The vertical displacement of the hanging mass, Δy, has the same magnitude as the horizontal displacement of the cart, Δx.

- The work done by gravity is also the negative of the change in potential energy of this system. Compare the amount of potential energy change, $\left|\Delta U_{\text{hanger}}\right|$, to the change in kinetic energy of the system by taking a percent difference.
- Calculate the energy "lost" in each trial.

Lab Report: The Work-Energy Theorem

Name: ____

DATA TABLES:

RUN	Mass of Cart (Includes the force sensor and a mass bar) M_{cart} *[kg]*	Hanging mass m_{hanger} *[kg]*	Total Mass of the System $M_{\text{cart}} + m_{\text{hanger}}$ *[kg]*
1			
2			
3			

	From the Position vs. Time Plot				From the Kinetic Energy vs. Time Plot			From the Force plot
	Initial x_o *[m]*	Final x_f *[m]*	Displacement Δx *[m]*		Initial K_o *[J]*	Final K_f *[J]*	Change in Kinetic Energy ΔK *[J]*	Mean value of Tension $T_{\text{on cart}}$
1				Cart:				
				Hanger:				
				System:				
2				Cart:				
				Hanger:				
				System:				
3				Cart:				
				Hanger:				
				System:				

ANALYSIS TABLES:

	THE WORK-ENERGY THEOREM		
	Area under the plot $W_{\substack{\text{TOTAL}\\ \text{on cart}}}$	Change in kinetic energy ΔK_{cart} (Transfer from Data Table.)	% Difference
1			
2			
3			

	THE LAW OF CONSERVATION OF MECHANICAL ENERGY			
	Work done by gravity $W_{\text{by gravity}} = mg \cdot \Delta x$ (This is $\left\lvert \Delta U_{\text{hanger}} \right\rvert$)	Total change in kinetic energy ΔK_{system} (Transfer from Data Table.)	% Difference	Energy 'lost' in the system
1				
2				
3				

Questions:

1. Discuss: How well the experimental verification of the Work-Energy Theorem matches the theory? What are sources of experimental uncertainty in this experiment?

2. Discuss: Why was some energy "lost" from this system? Where did it go?

3. Look at the data **for the hanging mass**: According to the theory, the change in kinetic energy of the hanging mass must be the total work done on the hanging mass. As discussed in the theory section, the hanging mass has two forces doing work on it: gravity and the tension of the string. If the total work is the work done by gravity plus the work done by the tension, calculate the work done by the tension for each case:

	$W_{\substack{\text{TOTAL}\\ \text{on hanger}}} = \Delta K_{\text{hanger}}$ (Copy from Data Table)	$W_{\text{by gravity}}$ (Copy from Data Table)	How much work was done by the tension on the hanger? $W_{\substack{\text{by T}\\ \text{on hanger}}}$	Tension on the hanger $T_{\text{on hanger}}$
1				
2				
3				

4. Is the work done by the tension on the vertical motion of the hanger positive or negative? Why?

5. Using the work done by the tension on the hanger, and the known vertical displacement of the hanger, calculate the tension on the string in each case. Is the value higher, lower or the same as the 'Mean' tension measured for the cart by the force sensor? Discuss.