Experiment P26: Rotational Inertia (Smart Pulley)

Concept	Time	SW Interface	Macintosh® file	Windows® file
rotational motion	45 m	500 or 700	P26 Rotational Inertia	P26_ROTA.SWS

EQUIPMENT NEEDED	
• *Science Workshop™ Interface*	• mass and hanger set
• Smart Pulley	• paper clips (for masses < 1 g)
• balance (for measuring mass)	• rotational apparatus
• calipers	• table clamp

PURPOSE

The purpose of this laboratory activity is to measure the rotational inertia of a ring experimentally and to compare this value to the theoretical value.

THEORY

Theoretically, the rotational inertia, I, of a ring is given by

$$I = \frac{1}{2} M(R_1^2 + R_2^2) \quad \text{Equation 1}$$

where M is the mass of the ring, R_1 is the inner radius of the ring, and R_2 is the outer radius of the ring.

To find the rotational inertia of the ring experimentally, a known torque is applied to the ring and the resulting angular acceleration is measured.

Since $\tau = I\alpha$

$$I = \frac{\tau}{\alpha} \quad \text{Equation 2}$$

where α is the angular acceleration and τ is the torque.

Now,

$$\tau = r \times F \quad \text{Equation 3}$$

where r is the distance from the center of the ring to the point where a force is applied, and F is the applied force. The value of $r \times F$ is $r F \sin ø$ where $ø$ is the angle between r and the direction of F, the applied force. The torque is maximum when r and F are perpendicular.

In this case, the applied force is the tension (T) in a string that is tied to a step pulley that is part of a rotational apparatus. The string is pulled by a hanging mass **m**. The value of r is the radius of the step pulley on the apparatus. The radius is perpendicular to the applied force (Tension).

Therefore, the torque is:

$$\tau = rT \quad \text{Equation 4}$$

Applying Newton's Second Law for the hanging mass, $\boldsymbol{m}$, results in:

$$\sum \boldsymbol{F} = \boldsymbol{ma} = \boldsymbol{mg} - \boldsymbol{T}$$

Solving for the tension in the string gives

$$\boldsymbol{T} = \boldsymbol{m}(\boldsymbol{g} - \boldsymbol{a})$$

The torque is:

$$\tau = \boldsymbol{rT} = \boldsymbol{rm}(\boldsymbol{g} - \boldsymbol{a}) \text{ Equation 5}$$

The linear acceleration $\boldsymbol{a}$ of the hanging mass is the tangential acceleration, $\boldsymbol{a_T}$, of the rotating apparatus.

The angular acceleration is related to the tangential acceleration as follows:

$$\alpha = \frac{\boldsymbol{a_T}}{\boldsymbol{r}} \text{ Equation 6}$$

Substituting Equation 5 and Equation 6 into Equation 2 gives:

$$\boldsymbol{I} = \frac{\tau}{\alpha} = \boldsymbol{rm}(\boldsymbol{g} - \boldsymbol{a}) \div \frac{\boldsymbol{a_T}}{\boldsymbol{r}} = \boldsymbol{rm}(\boldsymbol{g} - \boldsymbol{a})\frac{\boldsymbol{r}}{\boldsymbol{a_T}} = \frac{\boldsymbol{mgr}^2}{\boldsymbol{a_T}} - \boldsymbol{mr}^2 = \boldsymbol{mr}^2\left(\frac{\boldsymbol{g}}{\boldsymbol{a_T}} - \boldsymbol{1}\right)$$

The rotational inertia, $\boldsymbol{I}$, can be calculated from the tangential acceleration, $\boldsymbol{a_T}$.

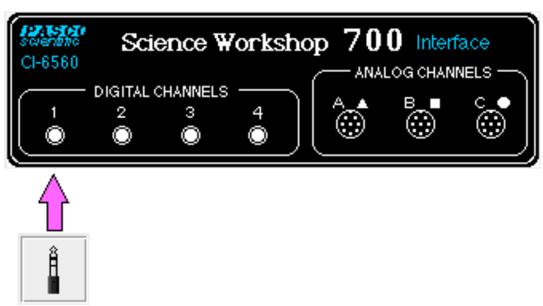
PROCEDURE

For this activity, the Smart Pulley measures the motion of a hanging mass that is connected by a string to a step pulley on the rotational apparatus. The *Science Workshop* program calculates and displays velocity versus time. The slope of the best fit line of velocity versus time is the value of acceleration.

PART I: Computer Setup

1. Connect the *Science Workshop* interface to the computer, turn on the interface, and turn on the computer.

2. Connect the Smart Pulley's stereo phone plug into Digital Channel 1 on the interface.

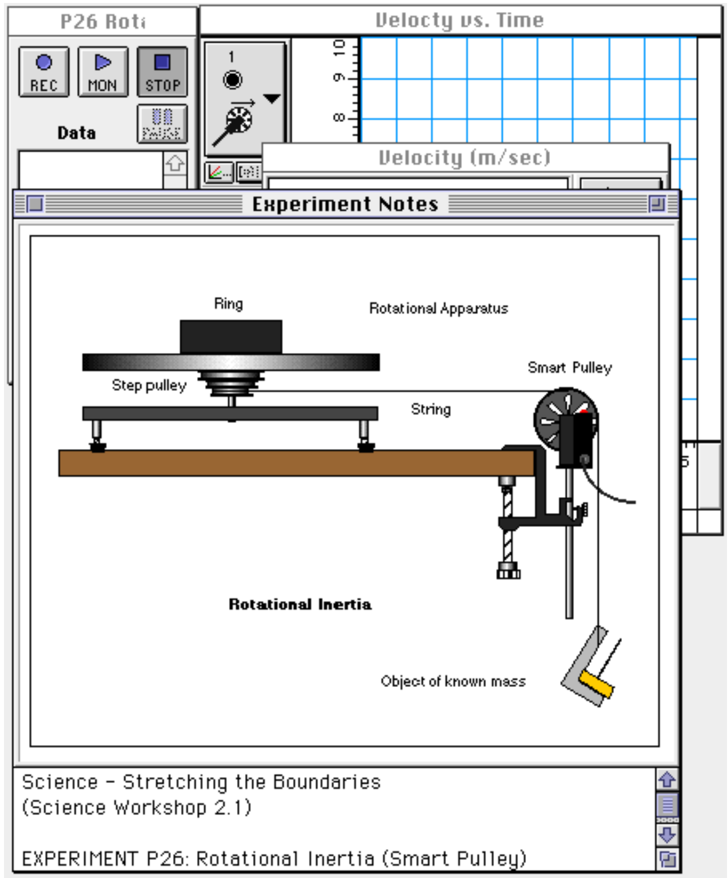


3. Open the *Science Workshop* document titled as shown:

Macintosh	Windows
P26 Rotational Inertia	P26_ROTA.SWS

- The document opens with a Graph display of Velocity (m/sec) versus Time (sec) and a Digits display of Velocity (m/sec).

- Note: For quick reference, see the Experiment Notes window. To bring a display to the top, click on its window or select the name of the display from the list at the end of the Display menu. Change the Experiment Setup window by clicking on the **Zoom** box or the **Restore** or **Maximize** button in the upper right hand corner of that window.



PART II: Sensor Calibration and Equipment Setup

- You do not need to calibrate the Smart Pulley.

1. Measure the diameter of the <u>smallest</u> step pulley on the main platter of the Rotational Apparatus. Calculate and record the radius of the step pulley in the Data section.

2. Measure the inside diameter of the ring. Calculate and record the inside radius of the ring in the Data section. Measure the outside diameter of the ring. Calculate and record the outside radius of the ring in the Data section.

3. Measure and record the mass of the ring in the Data section.

4. Attach the table clamp to the edge of a sturdy table. Mount the Smart Pulley's rod vertically in the clamp.

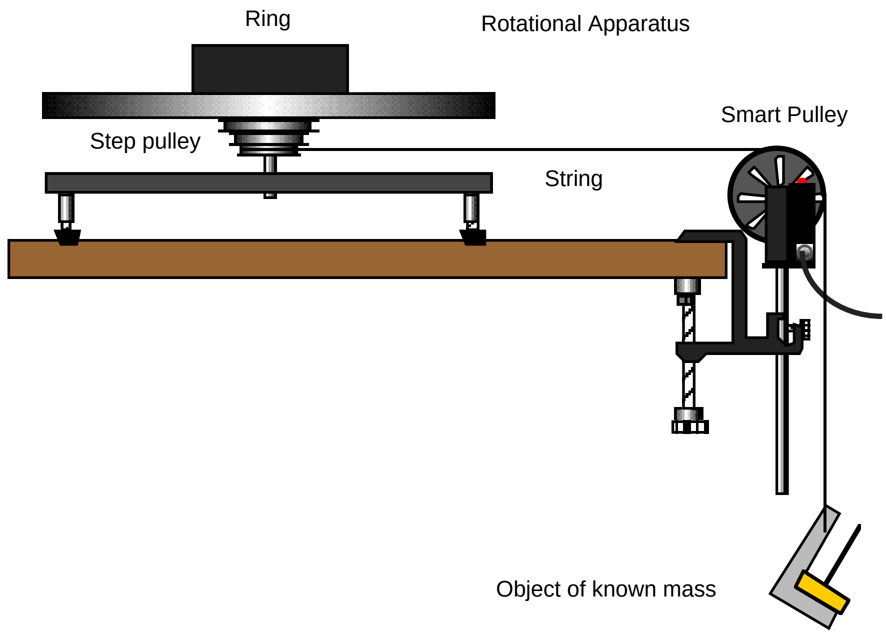
5. Place the Rotational Apparatus platform near the Smart Pulley. Carefully level the platform using a spirit level. Put the spindle in the bearing in the middle of the platform.

6. Use a string that is about 10 centimeters longer than the distance needed to reach from the spindle of the Rotational Apparatus over the Smart Pulley and down to the floor. Tie one end of the string to the edge of the smallest step pulley on the main platter of the Rotational Apparatus.

7. Mount the main platter on the spindle with the step pulley in the down position. Check whether the platter is level and adjust the platform if needed.

- Put the string in the groove of the Smart Pulley.

- Adjust the height of the Smart Pulley so the string from the step pulley is horizontal (level with the table top).



8. Place the ring on top of the main platter.

PART IIIA: Data Recording – Measurement of Friction, Ring & Main Platter

1. To compensate for friction, find out how much mass it takes on the end of the string to overcome kinetic friction and allow the mass to drop at a constant speed. This “friction mass” will be subtracted from the total mass used to accelerate the ring.

2. Attach a mass hanger on the end of the string. Wind the string around the lower step pulley by rotating the main platter until the mass hanger is raised almost to the Smart Pulley.

- If you are using a PASCO mass hanger, you can attach the string by wrapping the string three or four times in the notch.

3. Add just enough mass to the hanger so the mass falls with a constant, slow speed when you release the platter. Click the **Digits** display to make it active. Move it so you can see the display.

4. Click the **MON** button (
) to measure speed. Add or subtract mass from the hanger until the speed displayed in the Digits display is nearly constant.

- You can use individual paper clips to change the mass by small amounts.

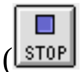
5. Click the **STOP** button (
) to end the measurement of friction.

6. Carefully measure and record the total mass on the end of the string in the Data section.

PART IIIB: Data Recording – Acceleration of the Ring & Main Platter

1. Measure the acceleration of the main platter and ring. Add about 50 g to the mass hanger attached to the string. Wind the string around the step pulley of the main platter by rotating the platter until the mass hanger is raised almost to the Smart Pulley. Hold the platter in place.

2. Click the **REC** button () to begin data recording and then release the main platter.

3. Click the **STOP** button () just before the mass hanger reaches the floor.

- **Run #1** will appear in the Data list in the Experiment Setup window.

4. Remove the mass hanger from the string. Measure the total mass of the hanger and record the value in the Data section.

PART IIIC: Data Recording – Measurement of Friction, Main Platter Alone

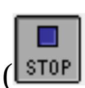
1. In Part IIIB, the ring was rotating together with the main platter. It is necessary to determine the acceleration, and the rotational inertia, of the main platter by itself. You will subtract the rotational inertia of the main platter from the total rotational inertia to find the rotational inertial of only the ring.

2. Remove the ring from the main platter. Wind the string around the step pulley of the main platter by rotating the platter until the mass hanger is raised almost to the Smart Pulley. .

3. Add just enough mass to the hanger so the mass falls with a constant, slow speed when you release the platter.

4. Click the **MON** button () to measure speed. Add or subtract mass from the hanger until the speed displayed in the Digits display is nearly constant.

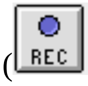
- You can use individual paper clips to change the mass by small amounts.

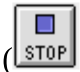
5. Click the **STOP** button () to end the measurement of friction.

6. Carefully measure and record the total mass on the end of the string in the Data section.

PART IIID: Data Recording – Acceleration of the Main Platter Alone

1. Measure the acceleration of the main platter alone. Add about 50 g to the mass hanger attached to the string. Wind the string around the step pulley of the main platter by rotating the platter until the mass hanger is raised almost to the Smart Pulley. Hold the platter in place.

2. Click the **REC** button () to begin data recording and then release the main platter.

3. Click the **STOP** button () just before the mass hanger reaches the floor.

• **Run #2** will appear in the Data list in the Experiment Setup window.

4. Remove the mass hanger from the string. Measure the total mass of the hanger and record the value in the Data section.

ANALYZING THE DATA

DATA TABLE 1: Dimensions

Item	Radius
Step Pulley (r)	m
Ring, inside radius (R_1)	m
Ring, outside dimension (R_2)	m

Mass of Ring (M) = _________ kg

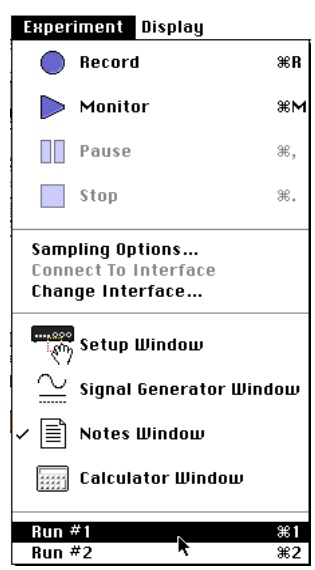
DATA TABLE 2: Mass

Run	Description	Friction Mass	Hanging Mass	Hanging mass - Friction mass (m)
#1	Main Platter & Ring	kg	kg	kg
#2	Main Platter	kg	kg	kg

1. Click the Graph to make it active. Click the **Statistics** button () to open the Statistics area on the right side of the Graph.

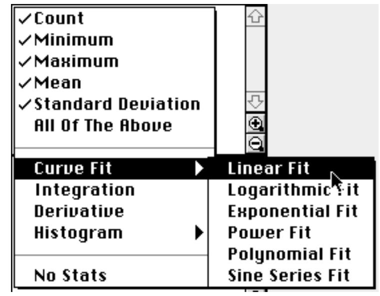
2. Click **Experiment** in the menu bar. Select **Run “1** from the end of the Experiment menu.

3. Click the **Autoscale** button () to rescale the Graph to fit the data.



4. Click the **Statistics Menu** button (Σ ▼). Select **Curve Fit, Linear Fit** from the Statistics Menu.

• The slope (coefficient **a2**) of the best fit line is the acceleration.



5. Record the slope (coefficient **a2**) as the acceleration for the Main Platter with Ring.

6. Click **Experiment** in the menu bar. Select **Run #2**. Record the new slope as the acceleration for the Main Platter alone.

DATA TABLE 3: Measured Accelerations (a_T)

Trial	Description	Acceleration
Run #1	Main Platter with Ring	m/sec/sec
Run #2	Main Platter	m/sec/sec

7. Calculate the experimental value of the rotational inertia, I, of the Main Platter with Ring using the measured acceleration **"a_T"**, the Step Pulley radius **"r"**, and the mass **"m"** that caused the apparatus to rotate.

$$I = \frac{\tau}{\alpha} = mr^2\left(\frac{g}{a_T} - 1\right)$$

8. Record the experimental value of the rotational inertia in the Data section.

9. Calculate the experimental value of the rotational inertia of the Main Platter using the measured acceleration **"a_T"**, the Step Pulley radius **"r"**, and the mass **"m"**. Record the value in the Data section.

10. Subtract the rotational inertia of the Main Platter from the value of the rotational inertia for the Main Platter with Ring to find the rotational inertia of the Ring alone. Record the experimental value for the Ring.

DATA TABLE 4: Rotational Inertia (I)

Trial	Description	Rotational Inertia
Run #1	Main Platter with Ring	kg m^2
Run #2	Main Platter	kg m2
	Ring alone	kg m^2

11. Calculate the theoretical value of the rotational inertia of Ring based on its dimensions (**R_1** and **R_2**) and mass (**M**). Record the value.

$$I = \frac{1}{2} M(R_1^2 + R_2^2)$$

Theoretical Rotational Inertia (I) of Ring = _________ kg m^2

12. Calculate the percent difference between the experimental value and the theoretical value for the rotational inertia of the Ring.

Percent Difference = _________ %

QUESTIONS

1. How does your experimental value compare to the theoretical value for rotational inertia?

2. What are some reasons that could account for any differences?

